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WAVE STATISTICS IN RANDOM MEDIA: A WAVE-KINETIC NUMERICAL APPROACH

FINAL REPORT

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The c	ieterminat	tion of the ir	radiance distri	bution of a	wave after m	ropagat	ion	
The determination of the irradiance distribution of a wave after propagation through a medium of random fluctuations of refractive index at scales large								
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compared to a wavelength has been an intractable problem. It has only been								
possible to calculate moments of the distribution asymptotically or numerically.								
In this effort, we try to simulate on computer the actual instantaneous random								
medium, and the propagation of waves through it by means of a phase-space								
function, the Wigner distribution function which carries information beyond								
ray crossings and caustics. The method uses an efficient algorithm for propaga-								
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propagated. Two canonical problems have been treated by the wave-kinetical method: free-space propagation and broadening of a collimated laser beam, and scattering of a plane wave by one Gaussian fluctuation of the refractive index. The results agree with what is expected. Simulation of a strong phase screen has been investigated, but the work is unfinished. The author's random-eddy model is still too unwieldy (in size) for the extended medium. Quasi-random models are currently being investigated.

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## STATEMENT OF THE PROBLEM:

Except for the explicit numerical calculation of several lower-order "moments" of the irradiance distribution of waves that have propagated through a continuous random medium such as turbulent air, it has not yet been feasible to carry out anything other than some asymptotic analytical calculations of quantities that shed light on the irradiance distribution. The nature of turbulent air is such that a size distribution of refractive-index fluctuations ranging from several millimeters to several meters (for ground-layer propagation phenomena) must fill one or more kilometers of pathlength in a narrow but not infinitesimal cone ending at the point of observation. The number of light rays needed for simulation is very large, and the distances are such that singularity problems (foci, caustics) must be overcome.

To overcome these singularity difficulties, a phase-space description of the waves is useful. The wave-kinetic approach consists of formulating a Wigner distribution function (WDF), for which an equation can be formulated, starting from the parabolic equation for the electric field. This equation has higher-order derivatives of the refractive index multiplying higher-order derivatives with respect to wavenumber of the WDF. In random media with scalelengths large comparable to the wavelength, and with weak local strengths, it can be shown that the higher-order-derivative terms are negligible, and that the remaining equation can still describe the diffractive effects so typical of long-range propagation through turbulent air. The approximated equation states that the WDF is conserved along ray trajectories, and its solution is found by the method of characteristics: the characteristics are simply the equations of geometrical optics! In spite of the geometrical-optics trajectories that carry the WDF along, the WDF still harbors information on diffraction, and is therefore only valid beyond the limit at which geometrical optics rays can meet and cross.

One other major aspect of the numerical method proposed for this effort is the use of a discretization of the initial WDF into a sum of Gaussian beamlets. If these beamlets have a sufficiently small halfwidth, then it follows that ray coordinates of rays at the "edges" of the beamlet, expressed with respect to the "centers" of the beamlets transform linearly from combinations at previous distances. The Gaussians contain exponents of quadratic combinations of those coordinates and hence remain Gaussian. The method concentrates on tracing the transformation of initial into end coordinates of each beamlet, than summing up all the Gaussian end beamlets, and producing from these (in trivial fashion) a final irradiance.

Another aspect of the method is to use a model of air turbulence developed previously by the author [de Wolf, Radio Sci.18, pp.138-142; 1983]. This model replaces a spectral decomposition by a sum of randomly moving Gaussians. The coefficients can be chosen to model given spectral characteristics. With such a model, one does not need to work in the wavenumber domain (with a Fourier transform).

## **SUMMARY:**

In a preliminary study, two canonical problems were used to test the model. The first is the free-space spreading of an initially collimated laser beam upon propagation. An analytical solution to this problem exists so that a comparison can be made. The second is the scattering of an initially collimated laser beam (that approximates a plane wave) by one spherical

refractive index fluctuation with Gaussian falloff in strength. If the 1/e length is small compared to the laser-beam diameter, then the latter approximates a plane wave well. Although no analytical solution to this problem exists, a brute force Monte-Carlo ray tracing method provided a check on the result.

Work on these two problems was reported in the first half-year report summarizing efforts through December 31, 1985: this report included a reprint of a paper [de Wolf and J-K.Pack, J.Opt.Soc.Am. A3, pp.532-535; 1986]. The numerical work itself was reported in the second half-year effort ending June 30, 1986: a pre-print of a later publication [J-K Pack and D.A.de Wolf, J.Opt.Soc.Am. A3, pp.1766-1771; 1986] was included.

In the second half of 1986, a concentrated effort was made to extend the work to a phase screen with strong fluctuations of phase at its exit. It became apparent that the eddy model of turbulence required excessive computer time to evaluate. To overcome that difficulty, several models of quasi-turbulence were developed. Rather than filling the phase screen with random eddies, it was decided to fill it with randomly-placed "nuclear" blocks of eddies. The exit angles of rays for given entrance angles into the block were calculated separately, once and for all. It was then feasible to calculate an intensity correlation function by averaging many random realizations of the quasi-turbulent phase screen. Agreement with estimated results by asymptotic methods culled from the Russian literature were quite poor, however.

We deemed it necessary to try to duplicate the work by Belousov and Yakushkin (see Progress Report December 31, 1986) to check for errors; this work appeared inpenetrable, and we derived the necessary approximations on our own for a Gaussian turbulence spectrum. Accurate intensity correlation curves were obtained that appeared to corroborate the Russian findings, to the extent that these were legible.

Since then -and up to February 1987- we have attempted alternative models for the phase screen; models that give a reasonable representation of the exit phase at the end plane of the screen. Our wave-kinetic method will then calculate the development beyond that. This work is still unfinished at the time of the contract's end -even after two extensions- but we intend to continue the work nevertheless. If worthy results come out of it, we intend to report those in some fashion consonant with the efforts funded under ARO in 1985-1986.